Coupled dust-lattice modes in complex plasmas

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The coupling between transverse and longitudinal dust-lattice modes due to the particle-wake interactions and vertical dust charge gradient is considered. It is shown that the dust-lattice waves can be subjected to a specific instability, the criterion for which has been derived. This instability can explain experimentally observed spontaneous excitation of vibrational modes in a plasma crystal when the pressure is decreased below a critical value.

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The fundamental problem for the understanding of the dynamical properties of various self-organized structures such as plasma crystals [1–5] is collective processes in complex plasmas. Plasma crystals sustain different collective modes, so-called dust-lattice (DL) waves, analogous to those in solid state physics [6]. The first to develop the theory for the longitudinal DL modes in a one-dimensional particle chain was Melandsø [7]. Experimentally, the propagation of these waves in a rf discharge has been clearly demonstrated by Homann *et al.* [8]. Later Vladimirov *et al.* [9] theoretically predicted the appearance of another type of DL perturbations, transverse DL waves, which have been experimentally observed in a one-dimensional particle chain by Misawa *et al.* [10]. Presently, the dust-lattice modes and their stability are extensively studied both theoretically and in many experiments using discharge plasmas in the laboratory and under microgravity conditions [11–15].

In the laboratory experiments, the highly negatively charged microparticles usually levitate in the sheath region of the horizontal negatively biased electrode where there is a balance between the gravitational and electrostatic forces acting in the vertical direction. There are a few characteristic features of such particle trapping. On the one hand, the complex plasma structures reveal the anisotropy due to the vertical supersonic ion flows that lead to the formation of an ion "wake" underneath the suspended microparticles [16,17]. The "wake effect" can induce instabilities in the plasma crystal due to the interaction of transverse and longitudinal DL modes [12]. On the other hand, the charge of the microparticles caused by the electron and ion currents onto the grain surfaces strongly depends on the vertical particle position in the sheath region. This introduces another anisotropy of the system related to the equilibrium charge gradient [11,18,19]. In the present paper, we study the influence of both factors: the particle-wake interaction and equilibrium charge inhomogeneity on coupling of the transverse and longitudinal DL modes and consider the conditions when the combination of these two factors results in a DL mode instability.

For DL waves in crystalline monolayers the model is that of a one-dimensional (1D) particle string, often invoked in theoretical considerations of DL perturbations [5,7,12]. This model allows two-dimensional motion, in the longitudinal (horizontal, along the string axis x) and transverse (vertical z) directions. The schematics of the string model is shown in Fig. 1. The horizontal string is formed by particles of the same mass M, separated by the average distance Δ along the x axis and carrying negative equilibrium charges. Contrary to the ordinary string model, we suppose that an equilibrium particle charge Q is characterized by a weak nonuniformity in the vertical z direction, namely, $Q(z)$. The equilibrium value of the charge is $Q_0=Q(z=0)$, where $z=0$ indicates the vertical equilibrium position of the string. For definiteness, Q_0 is supposed to be positive, thus denoting an absolute value of the dust charge. For a model of the ion wake, we refer to an earlier paper [12], where the excess positive charge in the wake (downstream from the particle along the vertical z axis) was considered as a pointlike effective charge $q_0=q(z=0)<Q_0$, located at a distance $l<\Delta$ beneath the particle (Fig. 1). When a particle n acquires a small vertical displacement around its equilibrium position z_n, the particle charge can be approximated as $Q_0 \to Q_n \simeq Q_0+Q_0' z_n$ with $Q_0'=(dQ/dz)_0$. The subscript 0 means that the derivative is taken at the equilibrium level $z=0$. It is reasonable to assume that the vertical displacements affect the values of the wake charge q in the same way, i.e., $q_0 \to q_n \simeq q_0+q_0' z_n$. Finally, we do not introduce any horizontal confinement of the system, but assume that there is a harmonic potential well in the vertical direction with eigenfrequency Ω_v, related to the confinement potential via $U_{\mathrm{conf}}=M\Omega_v^2 z^2/2$. The vertical frequency Ω_v is thus specified by the vertical electric field $E(z)$ and the equilibrium charge distribution $Q(z)$ as $\Omega_v^2 = -M^{-1}\partial(Q|E|)_0/\partial z$ [19].

Assuming that the interaction is confined between the closest neighbors, as is often the case in real lattices (usually the interparticle distance exceeds the screening length: $\Delta/\lambda_D \sim 1.5$–$3$), the equation of motion for the nth particle in the string can be written as

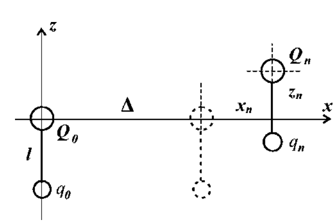

FIG. 1. The model of the particle string with a wake. Dashed lines correspond to the equilibrium particle position in the string.

$$\ddot{\mathbf{r}}_n + 2\gamma\dot{\mathbf{r}}_n = M^{-1}(\mathbf{F}_{n,n+1} + \mathbf{F}_{n,n-1} + \mathbf{F}_{n,\mathrm{conf}}), \quad (1)$$

where γ is a damping rate due to neutral gas friction [20], and $\mathbf{F}_{n,\mathrm{conf}} = -\partial U_{\mathrm{conf}}/\partial \mathbf{r}_n$ is the force due to the external confinement. Furthermore, the forces between the nodal nth and $(n\pm1)$th particles, $\mathbf{F}_{n,n\pm1}$, are determined by

$$\mathbf{F}_{n,n\pm1} = -Q_n(\partial\varphi_{n\pm1}/\partial\mathbf{r})_{r=r_n}. \quad (2)$$

The electrostatic potential of each particle $\varphi_{n\pm1}$ is given by the screened Debye potentials of the particle charge itself and its effective wake charge

$$\varphi_{n\pm1}(z) = \frac{Q_{n\pm1}\exp(-\Delta_{Q\pm}/\lambda_D)}{\Delta_{Q\pm}} - \frac{q_{n\pm1}\exp(-\Delta_{q\pm}/\lambda_D)}{\Delta_{q\pm}}. \quad (3)$$

Here $\Delta_{Q\pm}$ denotes the distance between adjacent particles, and $\Delta_{q\pm}$ is the distance between the nodal (n) particle and the effective wake charge of the adjacent $(n\pm1)$ grains. Introducing the particle displacements in the horizontal and vertical directions of the nodal nth particle $\{x_n, z_n\}$ and similarly for the right and left neighbor grains $\{x_{n\pm1}, z_{n\pm1}\}$, the interparticle distances can be written as $\Delta_{Q\pm} = \sqrt{(\Delta+\delta x_{n\pm1})^2 + \delta z_{n\pm1}^2}$, and $\Delta_{q\pm} = \sqrt{(\Delta+\delta x_{n\pm1})^2 + (l+\delta z_{n\pm1})^2}$, respectively. Both of these distances depend on the relative displacement with respect to the right, $\delta x_{n+1} = x_{n+1} - x_n$, and to the left, $\delta x_{n+1} = x_n - x_{n-1}$, neighbor.

Since the particles are sufficiently strongly coupled, we can safely assume that they only have small displacements $(|x_{n\pm1} - x_n|, |z_{n\pm1} - z_n| \ll \Delta, l)$. Combining relations (2) and (3) with the equation of motion (1) and expanding the right hand side of Eq. (1) around its equilibrium gives, to first order in x_n and z_n,

$$\ddot{x}_n + 2\gamma\dot{x}_n = \Omega_{xx}^2(x_{n+1} + x_{n-1} - 2x_n) + \Omega_{xz}^2(z_{n+1} - z_{n-1}), \quad (4)$$

$$\ddot{z}_n + 2\gamma\dot{z}_n = -\Omega_v^2 z_n - \Omega_{zz}^2(z_{n+1} + z_{n-1} - 2z_n) - \Omega_{Qq}^2(z_{n+1} + z_{n-1} + 2z_n) + \Omega_{zx}^2(x_{n+1} - x_{n-1}). \quad (5)$$

In general, the elements of these mode equations, Ω_{ij}^2, are given by bulky expressions. For simplification, we will use the condition $l < \Delta$, which is relevant for a strongly coupled complex plasma. Indeed, in laboratory complex plasmas the distance l never exceeds the screening length λ_D [12], which in turn is smaller than Δ (typically $\Delta/\lambda_D \sim 1.5-3$). Therefore we take $(l/\Delta)^2 \ll 1$ and neglect terms $O(l^2/\Delta^2)$. The squared frequencies then become

$$\Omega_{xx}^2 = (1-\tilde{q})\Psi(\kappa)\Omega_0^2 = \Omega_{\parallel}^2, \quad (6)$$

$$\Omega_{zz}^2 = (1-\tilde{q})\varphi(\kappa)\Omega_0^2 = \Omega_{\perp}^2, \quad (7)$$

$$\Omega_{Qq}^2 = \epsilon\tilde{q}\tilde{l}\varphi(\kappa)\Omega_0^2, \quad (8)$$

$$\Omega_{zx}^2 = \tilde{q}\tilde{l}\Phi(\kappa)\Omega_0^2, \quad (9)$$

$$\Omega_{xz}^2 = \Omega_{zx}^2 - (\varepsilon - \epsilon\tilde{q})\varphi(\kappa)\Omega_0^2. \quad (10)$$

Here the characteristic DL frequency Ω_0 is defined through

$$\Omega_0^2 = \frac{Q_0^2}{M\Delta^3}\exp(-\kappa), \quad (11)$$

$\kappa = \Delta/\lambda_D$ denotes the lattice parameter, $\tilde{q} = q_0/Q_0$ is the ratio of the effective wake charge to the equilibrium particle charge, and $\tilde{l} = l/\Delta$. Moreover, we have introduced the notations $\varphi(\kappa) = (1+\kappa)$, $\Psi(\kappa) = 1 + \varphi^2(\kappa)$, $\Phi(\kappa) = \varphi(\kappa) + \Psi(\kappa)$, $\varepsilon = Q_0'\Delta/Q_0$, and $\epsilon = q_0'\Delta/q_0$. The small parameters ε and ϵ are of the same order of $\varepsilon \sim \Delta/L_Q$, $\epsilon \sim \Delta/L_q$, where $L_Q \sim Q(z)/[\partial Q(z)/\partial z]$ and $L_q \sim q(z)/[\partial q(z)/\partial z]$ are the characteristic scales of the particle charge and wake charge nonuniformity.

While the frequencies of longitudinal $(\Omega_{\parallel})$ and transverse $(\Omega_{\perp})$ DL modes include only the wake effect, which might decrease the main frequency of the DL waves via the factor $(1-\tilde{q})$, the eigenfrequency Ω_v contains the vertical charge and field gradients through

$$\Omega_v^2 = -Q_0M^{-1}(\varepsilon|E|_0/\Delta + |E|_0'), \quad (12)$$

and thus Ω_v^2 generally can be either positive or negative for different particle positions in the sheath region: it is expected that $\Omega_v^2 > 0$ in most of the sheath—in the presheath and below the sheath edge [19]. However, Ω_v^2 can become negative in a narrow region of the sheath just above the lower electrode, with a possible resultant instability of the vertical motion of particles (see region I in Fig. 3, Ref. [19]). Typically, the particles are trapped near the sheath edge, so we can assume from the outset that $\Omega_v^2 > 0$ to ensure that the transverse mode is not aperiodic unstable.

The frequency Ω_{Qq} is a result of the combined effect of the wake and equilibrium charge gradient on the transverse wave mode. We generally have the case that $\epsilon < 1$, $\tilde{q} < 1$, and $\tilde{l} < 1$, so the contribution of the hybrid term Ω_{Qq}^2 can be significantly smaller than Ω_{zz}^2.

Finally, the squared frequency Ω_{zx}^2 characterizes the coupling between transverse and longitudinal waves induced by the particle-wake interaction, while Ω_{xz}^2 describes the coupling due to the joint influence of the wake and equilibrium charge gradient on the DL modes. The latter element can be simplified to

$$\Omega_{xz}^2 \simeq \Omega_{zx}^2 - \varepsilon\varphi(\kappa)\Omega_0^2. \quad (13)$$

The reason for the asymmetry in the Ω_{ij}^2 terms is that the equilibrium charge is predominantly a function of the vertical position. Putting $\varepsilon = 0$ immediately restores the symmetry in the elements Ω_{ij}^2 and corroborates the results for the coupling of DL modes due to the ion wakes, obtained in Ref. [12].

Considering all particle displacements in Eqs. (4) and (5) to be $x_n, z_n \propto \exp[-i(\omega t - nk\Delta)]$, where the wave number k obeys $-\pi \leq k\Delta \leq \pi$, we obtain the dispersion relation

$$\left(\omega^2 + 2i\gamma\omega - 4\Omega_\parallel^2 \sin^2 \frac{k\Delta}{2}\right) \times \left(\omega^2 + 2i\gamma\omega - \Omega_v^2 + 4\Omega_\perp^2 \sin^2 \frac{k\Delta}{2} - 4\Omega_{Qq}^2 \cos^2\frac{k\Delta}{2}\right) + 4\Omega_{zx}^2[\Omega_{zx}^2 - \varepsilon(1+\kappa)\Omega_0^2]\sin^2 k\Delta = 0, \quad (14)$$

which combines the longitudinal (x) and transverse vertical (z) DL modes. We focus on the situation, when the effective wake charge satisfies the inequality $\tilde{q} < \Delta^2/lL_Q$ leading to $\Omega_{zx}^2 < \varepsilon(1+\kappa)\Omega_0^2$. This condition appears appropriate for many laboratory complex plasma experiments. Indeed, the values Δ usually exceed a few hundred micrometers, while the characteristic scales for the equilibrium charge variation are supposed to be of the order of the sheath size. Taking typical parameters $\Delta \sim 5\times 10^2$ μm, $L_Q \sim 0.5$ cm, and assuming for the wake scale length $l \sim 0.1\Delta$, we then obtain $\tilde{q} < 1$. Therefore the element Ω_{xz}^2 can be approximated by

$$\Omega_{xz}^2 \simeq -\varepsilon(1+\kappa)\Omega_0^2. \quad (15)$$

Note that now Ω_{xz}^2 will be either positive or negative depending on the sign of the value $\varepsilon \propto (\partial Q/\partial z)_0$ ($\varepsilon<0$ or $\varepsilon>0$) at the equilibrium position ($z=z_0$) of the particle string.

The frequency responsible for the DL mode coupling in Eq. (14) is given by

$$\Omega_{\text{coup}}^2 = |\Omega_{xz}|\Omega_{zx}. \quad (16)$$

This coupling is relatively weak, since $\Omega_{\text{coup}}^2/\Omega_0^2 = 2|\Omega_{xz}|\Omega_{zx}/\Omega_0^2 \sim \sqrt{\tilde{q}l/L_Q} \leqslant (\Delta/L_Q) < 1$ and hence can play a role only in the vicinity of the intersection point of the two decoupled branches. For a positive value Ω_v^2, satisfying

$$4\Omega_\perp^2 < \Omega_v^2 < 4(\Omega_\parallel^2 + \Omega_\perp^2), \quad (17)$$

the noninteracting DL modes intersect at the point (ω_0, k_0) given by

$$\omega_0 = \frac{\Omega_v}{\sqrt{1+s^2}}, \quad k_0\Delta = 2 \arcsin \frac{\omega_0}{2\Omega_\parallel}, \quad (18)$$

where $s^2 = \Omega_\perp^2/\Omega_\parallel^2$. Far from the intersection point, or in the case when condition (17) is not satisfied and hence DL modes do not intersect at all, the longitudinal (x) mode dispersion is almost not affected by charge gradient terms and has the typical form [12]

$$\text{Re}\,\omega \simeq 2\Omega_\parallel \sin\frac{k\Delta}{2}, \quad \text{Im}\,\omega \simeq -\gamma, \quad (19)$$

while the transverse (z) DL mode is slightly modified due to the small hybrid frequency Ω_{Qq}, namely,

$$(\text{Re}\,\omega)^2 \simeq \Omega_v^2 - 4\Omega_\perp^2 \sin^2\frac{k\Delta}{2} + 4\Omega_{Qq}^2\cos^2\frac{k\Delta}{2}, \quad \text{Im}\,\omega \simeq -\gamma \quad (20)$$

The coupling term results in further small corrections $\sim O(\Omega_{\text{coup}}^4/\Omega_0^4)$ to the dispersion for both waves.

As k approaches k_0, the longitudinal (19) and transverse (20) modes can be significantly modified. The wave behavior can be understood by considering the solutions of the dispersion relation (14) in the vicinity of the intersection point (18) graphically. We normalize (14) by introducing the squared frequency $f=\omega^2/\Omega_0^2$, wave number $K=k\Delta/2$, $\alpha=\Omega_v/\Omega_0$, and the coupling parameter $\beta=4\tilde{q}\tilde{l}\varepsilon\Phi(\kappa)\varphi(\kappa)$. Assuming for simplicity $\gamma=0$ and $s\sim 1$ the dimensionless version of Eq. (14) becomes

$$(f - 4\sin^2 K)(f - \alpha^2 + 4\sin^2 K) - \beta \sin^2 2K = 0. \quad (21)$$

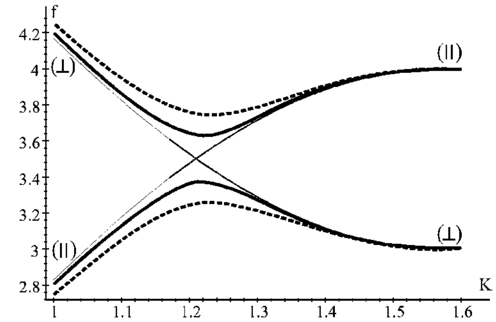

FIG. 2. The normalized solutions of the dispersion relation (14), describing the coupling between the transverse ($\perp$) and longitudinal ($\parallel$) DL modes in the case $\varepsilon>0$; $f=\omega^2/\Omega_0^2$ vs $K=k\Delta/2$ is shown for $\alpha=2.5$, $\beta=0.05$ (solid line), and $\beta=0.15$ (dashed lines). Decoupled modes [($\perp$) and ($\parallel$)] are represented by thin lines.

Figure 2 shows the behavior of $f=\omega^2/\Omega_0^2$ as a function of wave number K when $\varepsilon \propto (\partial Q/\partial z)_0 > 0$. Putting for definiteness $\alpha=2.6$, we present the solutions of Eq. (21) for two different values of the coupling parameter $\beta=0.05$ (solid line) and 0.15 (dashed line), respectively. The ordinary transverse ($\perp$) and longitudinal ($\parallel$) DL waves, corresponding to the solutions of Eq. (21) for $\beta=0$, are indicated by thin lines. As can be seen from Fig. 2, three curves are close to an appropriate ordinary mode ($\perp$) or ($\parallel$) only in the long-wavelength ($K \ll K_0 = k_0\Delta/2$) and short-wavelength ($K \gg K_0$) regimes. The coupled curves ($\beta \neq 0$) start deviating from the decoupled solution ($\beta=0$) in the vicinity of the intersection point (18). Finally, the long-wavelength transverse branch ($K \ll K_0$) joins with the short-wavelength longitudinal DL wave ($K \gg K_0$) and vice versa. An instability is not possible at all. In other words, the function $f(K)$ corresponds to a confluence of both transverse and longitudinal dispersion curves in such a way that there is a considerable frequency gap in the vicinity of the intersection point, in which both DL modes are evanescent. This type of reconnection between the two modes of DL waves can be considered as a kind of linear mode conversion.

Figure 3 represents the DL mode reconnection $f(K)$ in the other limiting case, when the initial particle string is situated at a level where $\varepsilon \propto (\partial Q/\partial z)_0 < 0$. As previously, two values of $\beta=-0.05$ (solid line) and -0.15 (dashed line) are considered. It turns out that now a confluence of the transverse ($\perp$) and longitudinal ($\parallel$) branches occurs separately in the long-wavelength ($K<K_0$) and short-wavelength ($K>K_0$) ranges, thus leading to a gap in the wave number domain, and a resultant instability of the hybrid mode. The growth of the DL perturbations becomes physically possible at the expense

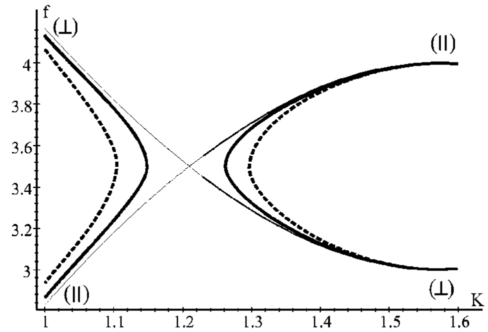

FIG. 3. The normalized solutions of the dispersion relation (14), describing the coupling between the transverse $(\perp)$ and longitudinal $(\parallel)$ DL modes in the case $\varepsilon<0$; $f=\omega^2/\Omega_0^2$ vs $K=k\Delta/2$ is shown for $\alpha=2.5$, $\beta=-0.05$ (solid line), and $\beta=-0.15$ (dash lines). Decoupled modes $[(\perp)$ and $(\parallel)]$ are represented by thin lines.

of the externally pumped in the discharge chamber energy that creates the equilibrium dust charge distribution $Q(z)$.

To get the growth rate of this instability we expand Eq. (14) around (ω_0,k_0),

$$\left(\omega-\omega_0+i\gamma-\Omega_\parallel\cos\frac{k_0\Delta}{2}(k-k_0)\Delta\right)\left(\omega-\omega_0+i\gamma+\frac{\Omega_\perp^2}{\Omega_\parallel}\times\cos\frac{k_0\Delta}{2}(k-k_0)\Delta\right)\simeq-\frac{\Omega_{\rm coup}^4}{\omega_0^2}\sin^2 k_0\Delta. \qquad (22)$$

To lowest order in $|\omega-\omega_0|/\omega_0\sim\Omega_{\rm coup}^2/\Omega_v^2\ll 1$,

$$\mathrm{Re}\,\omega\simeq\omega_0,\quad \mathrm{Im}\,\omega\simeq-\gamma+\frac{\Omega_{\rm coup}^2}{\omega_0}\sin k_0\Delta,$$

indicating that this hybrid mode is unstable, when the neutral gas friction is sufficiently weak, namely,

$$\gamma<\frac{\Omega_{\rm coup}^2}{\Omega_\parallel}. \qquad (23)$$

Note that the instability occurs in the narrow wave number domain $k_1<k_0<k_2$, with k_1 and k_2 being defined by the combined effect of the ion wake and dust charge gradient on the DL modes through

$$k_{1,2}\simeq k_0\left(1\pm\frac{\Omega_{\rm coup}^2}{(\Omega_\parallel^2+\Omega_\perp^2)\cos k_0\Delta}\right). \qquad (24)$$

In laboratory experiments ($Q_0\sim 10^3 e,\Delta\sim 5\times 10^2$–$10^3$ μm), the typical value of the DL frequency (11) is $\Omega_0\sim 10^2$ s^{-1} [3,8], while the characteristic scales for the equilibrium charge variation are $L_Q\sim 0.5$ cm [21], thus giving $\Omega_{\rm coup}^2\sim(\Delta/L_Q)\Omega_0^2\sim 10^3$ s^{-2}. Therefore, the criterion for the DL mode instability (23) will be satisfied for DL waves of frequencies $\omega_0\sim 10^2$ s^{-1} at a neutral gas pressure below tens of pascals ($\gamma\sim 10$ s^{-1}, $p\lesssim 10$ Pa). Note that neutral gas damping quenches the DL mode instability associated with pure particle-wake interactions already at gas pressures $p\leqslant 1$ Pa [12], while the considered instability can be of importance at a gas pressure of ~ 10 Pa.

To summarize, we have demonstrated that the combined effect of ion focusing and vertical dust charge nonuniformity leads to an interaction between transverse and longitudinal DL modes. Moreover, this can lead to an instability in a horizontal chain (or monolayer) of the microparticles if the particle levitation occurs in that part of the sheath where the absolute value of the equilibrium charge is reduced (e.g., region II in Fig. 3, Ref. [19]) and provided neutral gas damping below a certain threshold. It is possible, therefore, that the experimentally observed spontaneous excitations of DL modes that occur in complex plasmas when the pressure is decreased below a critical value [15,22] (e.g., in [22], $p\leqslant 4.5$ Pa) are a manifestation of the discovered instability.

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